

EET165 Lecture #2

- 1) A numbering system is used to count and keep track of how many of something you have. We have 10 fingers so our numbering system is based on 10. So, we say that we have a base 10 numbering system. No matter what base you use, the numbering system will accurately keep count of the items. We will be covering Base 10, Base 2, and Base 16.

BASE 10: The decimal numbering system is the numbering system that humans are used to using; it is a Base 10 numbering system. Base 10 means that there are only 10 symbols used in the whole numbering system, they are [0,1,2,3,4,5,6,7,8,9]. When counting in base 10 each location in a number is 10 times larger than the location to its right. For example, the number $5,802_{10}$:

5	8	0	2
(1000's place)	(100's place)	(10's place)	(1's place)

The number 5802 is made up of 2 ones, 0 tens, 8 hundreds, and 5 thousands. The location of a digit has meaning. The far-right digit is always the ones place. Each place to the left is ten times larger than the previous place. To go from the ones place to the tens place, *multiply by 10*; to go from the tens place to the hundreds place, *multiply by 10*; and to go from the hundreds place to the thousands place, *multiply by 10*. When counting in base 10 eventually all of the symbols are used up. When this happens, the next place is incremented and the previous place starts over.

Pretend you are putting eggs in a carton, but the cartons can only hold 10 eggs. You start with an empty carton (0 eggs) and start filling it up. When you get to the end (put in the 10th egg) you close the carton and put it on your left side. You then would have to grab a new empty carton to start filling again. After filling for a while, you might have 5 full containers and a 6th container with only 2 eggs in it. That means you have 5 groups of ten and 2 ones, this is written as 52.

BASE 2: The binary numbering system is a base 2 numbering system. This means that there are only two symbols used in the whole numbering system, they are [0,1]. When counting in BASE 2 the far-right digit is the ones place. Every place you move to the left is 2 times larger than the previous place. For example, the number 1011_2 :

1	0	1	1
(8's place)	(4's place)	(2's place)	(1's place)

The number 1011_2 is made up of 1 one, 1 two, 0 fours, and 1 eight. Each place is two times larger than the place to the right. To go from the ones to the twos, *multiply by 2*; to go from the twos to the fours, *multiply by 2*; and to go from the fours to the eights, *multiply by 2*. When counting in base 2 and all of the symbols are used up, the next place is incremented and the previous place starts over (just like in base 10).

Pretend you are putting eggs in a carton, but the cartons can only hold 2 eggs. You start with an empty carton (0 eggs) and start filling it up. When you get to the end (put in the 2nd egg) you close the carton and put it on your left side. You then would have to grab a new empty carton to start filling again. If you put one egg in a second carton, you would have 11_2 eggs. That means you have 1 box of 2 eggs and 1 single egg – this means you have 3 eggs total.

BASE 16: The hexadecimal numbering system is a base 16 numbering system. This means that there are sixteen symbols used in the whole numbering system, they are [0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F]. When counting in hex (Base 16) the far-right digit is the ones place. Every place you move to the left is 16 times larger than the previous place. For example the number $13AC_{16}$:

1	3	A	C
(4096's place)	(256's place)	(16's place)	(1's place)

The number $13AC_{16}$ is made up of “C” ones, “A” sixteens, “3” two-fifty-sixes, and “1” four-thousand-ninety-sixes. (NOTE: C is what a human thinks of as 12 and A is what a human thinks of as 10.) Each place is sixteen times larger than the place to the right. To go from the ones to the sixteens, *multiply by 16*; to go from the sixteens to the two-fifty-sixes, *multiply by 16*; and to go from the two-fifty-sixes to the four-thousand-ninety-sixes, *multiply by 16*. When counting in base 16 and all of the symbols are used up, the next place is incremented and the previous place starts over.

Pretend you are putting eggs in a carton that holds 16 eggs. You start with an empty carton (0 eggs) and start filling it up. When you get to the end (put in the 16th egg) you close the carton and put it on your left side. You then would have to grab a new empty carton to start filling again. At the end you might have 10 full cartons (written as A) and 12 of the 16 spaces filled in the next carton (written as C) so you would have AC_{16} eggs. As a human you might think of this as $(10 * 16 \text{ eggs}) + 12 \text{ eggs}$ which would equal 172 eggs.

The table below shows the first 16 numbers in Base 10, Base 2, and Base 16.

<u>Base 10</u>	<u>Base 2</u>	<u>Base 16</u>	<u>Base 10</u>	<u>Base 2</u>	<u>Base 16</u>
0	0000	0	8	1000	8
1	0001	1	9	1001	9
2	0010	2	10	1010	A
3	0011	3	11	1011	B
4	0100	4	12	1100	C
5	0101	5	13	1101	D
6	0110	6	14	1110	E
7	0111	7	15	1111	F

- 2) If you have a value in any base, you can find the base 10 value using the same procedure. This procedure is done through multiplication.

Base 2 to Base 10 Conversion: To convert from base 2 to base 10, each space represents a power of two. The far left is the one's place and each space to the right is 2 times larger. The value in that position tells you how many of that value you have. To convert back to base 10, multiply the value in each position by the weight of that position and then sum the values.

Question: What is 1101011_2 in Base10 ?

$$\begin{array}{r}
 1101011_2 \\
 \begin{array}{l}
 \text{---} 1 * 1 = 1 \\
 \text{---} 1 * 2 = 2 \\
 \text{---} 0 * 4 = 0 \\
 \text{---} 1 * 8 = 8 \\
 \text{---} 0 * 16 = 0 \\
 \text{---} 1 * 32 = 32 \\
 \text{---} 1 * 64 = 64 \\
 \hline
 107_{10}
 \end{array}
 \end{array}$$

Or just simply 107.

For practice show that:

$$(101110_2 = 46_{10}) \quad (111101_2 = 61_{10}) \quad (10011_2 = 19_{10}) \quad (100101_2 = 37_{10})$$

Base 16 to Base 10 Conversion: To convert from base 16 to base 10, each space represents a power of sixteen. The far right is the one's place and each space to the left is 16 times larger. To convert back to base 10, multiply the value in each position by the

$$\begin{array}{r}
 1A4_{16} \\
 \begin{array}{l}
 \text{---} 4 * 1 = 4 \\
 \text{---} A * 16 = 160 \\
 \text{---} 1 * 256 = 256 \\
 \hline
 420_{10}
 \end{array}
 \end{array}$$

weight of that position and then sum the values.

Question: What is $1A4_{16}$ in base 10?

Or just simply 420.

For practice show that:

$$(5E_{16} = 94_{10}) \quad (4D_{16} = 77_{10}) \quad (6C_{16} = 108_{10}) \quad (2E_{16} = 46_{10})$$

- 3) If you have a value in base 10, you can find the equivalent value in any base using the same procedure. This procedure is done through division.

Base 10 to Base 2 Conversion: To go from base 10 to base 2 is a bit more difficult. Exiting base 10 is done by successive division. By example, we will convert the number 11_{10} to binary.

A $\begin{array}{r} 5 \text{ (R 1)} \longrightarrow (1) \\ 2 \overline{) 11} \end{array}$	B $\begin{array}{r} 2 \text{ (R 1)} \longrightarrow (1) \\ 2 \overline{) 5} \text{ (R 1)} \longrightarrow (1) \\ 2 \overline{) 11} \end{array}$
C $\begin{array}{r} 1 \text{ (R 0)} \longrightarrow (0) \\ 2 \overline{) 2} \text{ (R 1)} \longrightarrow (1) \\ 2 \overline{) 5} \text{ (R 1)} \longrightarrow (1) \\ 2 \overline{) 11} \end{array}$	D $\begin{array}{r} 0 \text{ (R 1)} \longrightarrow (1) \\ 2 \overline{) 1} \text{ (R 0)} \longrightarrow (0) \\ 2 \overline{) 2} \text{ (R 1)} \longrightarrow (1) \\ 2 \overline{) 5} \text{ (R 1)} \longrightarrow (1) \\ 2 \overline{) 11} \end{array}$ 

In block A: 11 is divided by 2; the answer is 5 Remainder 1.
So cut off the remainder, and make it a Binary 1.

In block B: 5 is divided by 2; the answer is 2 Remainder 1.
So cut off the remainder, and make it a Binary 1.

In block C: 2 is divided by 2; the answer is 1 Remainder 0.
So cut off the remainder, and make it a Binary 0.

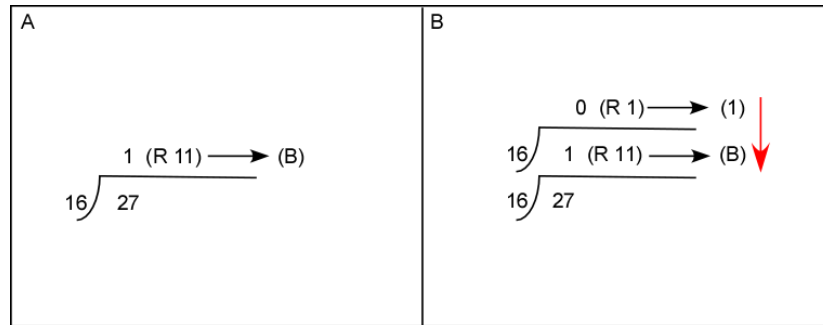
In block D: 1 is divided by 2; the answer is 0 Remainder 1.
So cut off the remainder, and make it a Binary 1.

When you have zero left, it is time to stop.
The answer is 1011_2 as shown in block D.

For practice show that:

$$(40_{10} = 101000_2) \quad (63_{10} = 111111_2) \quad (23_{10} = 10111_2) \quad (36_{10} = 100100_2)$$

Base 10 to Base 16 Conversion: To go from base 10 to base 16, the same method, called "successive division" is performed. This time you divide by 16 instead of 2. By example, we will convert the number 27_{10} to hex.



In block A: 27 is divided by 16; the answer is 1 Remainder 11.
So cut off the remainder, (11 is B in hex) and the answer is Hex B.

In block B: 1 is divided by 16; the answer is 0 Remainder 1.
So cut off the remainder, and the answer is Hex 1.

When you have zero left, it is time to stop.
The answer is $1B_{16}$ as shown in block B.

For practice show that:

$$(40_{10} = 28_{16})$$

$$(63_{10} = 3F_{16})$$

$$(23_{10} = 17_{16})$$

$$(236_{10} = EC_{16})$$

- 4) Converting Base 16 to Base 2 and Base 2 to Base 16 is a lot easier than you might think. You can go between these two bases in your head. You just need to memorize the chart that was covered earlier.

<u>Base 10</u>	<u>Base 2</u>	<u>Base 16</u>	<u>Base 10</u>	<u>Base 2</u>	<u>Base 16</u>
0	0000	0	8	1000	8
1	0001	1	9	1001	9
2	0010	2	10	1010	A
3	0011	3	11	1011	B
4	0100	4	12	1100	C
5	0101	5	13	1101	D
6	0110	6	14	1110	E
7	0111	7	15	1111	F

Base 16 to Base 2 Conversion: By far the simplest conversion is between base 2 and base 16 and base 16 to base 2. The reason it is so easy is because of a mathematical trick based on $2^4 = 16$. Because 16 is an integer power of 2, you can group four bits together and make them a single hex digit with no math at all. All you need to do is memorize the given table.

Question: What is $1A4_{16}$ in base 2?

Answer:

1 becomes 0001 A becomes 1010 4 becomes 0100

Or just simply: $\cancel{000}110100100_2 = 110100100_2$

For practice show that:

$$(5E_{16} = 01011110_2) \quad (4D_{16} = 01001101_2) \quad (6C_{16} = 01101100_2) \quad (2E_{16} = 00101110_2)$$

Base 2 to Base 16 Conversion: Just reverse the process and group the bits into groups of four and substitute the appropriate hex character.

Question: What is 11010101001_2 in base 16?

Answer: (From right to left)

1001 becomes 9
 1010 becomes A
 110 (or 0110) becomes 6

Or just simply: $6A9_{16}$

For practice show that:

$$(10101010_2 = AA_{16}) \quad (1101111_2 = 6F_{16}) \quad (111000111_2 = 1C7_{16}) \quad (11000_2 = 18_{16})$$

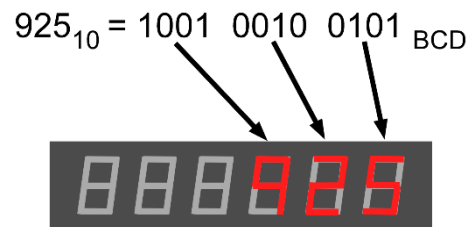
- 5) Codes are not numbering systems. They are made up by a person and are not based in math. Some codes include ASCII, BCD, and Gray code.

ASCII – The American Standard Code of Information Interchange. When you press a value on a keyboard you (as a human) know what a letter (such as A) means. But a computer is a thing. It runs a binary where the zero is a ground and one is 5 volts. When computers were created, engineers needed a way to store all the letters, numbers, and symbols in a way the computer could save it. ASCII is just a list of “This letter is stored as this binary value” chart.

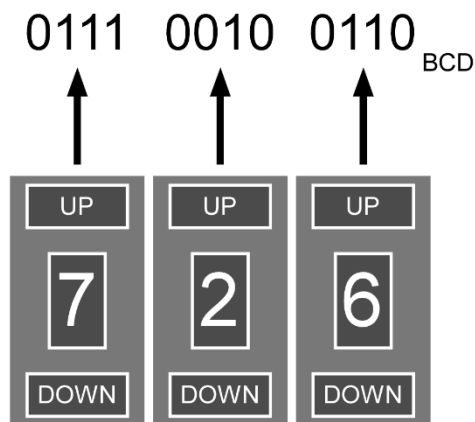
Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char
0	00	[NULL]	32	20	[SPACE]	64	40	@	96	60	`
1	01	[START OF HEADING]	33	21	!	65	41	A	97	61	a
2	02	[START OF TEXT]	34	22	"	66	42	B	98	62	b
3	03	[END OF TEXT]	35	23	#	67	43	C	99	63	c
4	04	[END OF TRANSMISSION]	36	24	\$	68	44	D	100	64	d
5	05	[ENQUIRY]	37	25	%	69	45	E	101	65	e
6	06	[ACKNOWLEDGE]	38	26	&	70	46	F	102	66	f
7	07	[BELL]	39	27	'	71	47	G	103	67	g
8	08	[BACKSPACE]	40	28	(	72	48	H	104	68	h
9	09	[HORIZONTAL TAB]	41	29	)	73	49	I	105	69	i
10	0A	[LINE FEED]	42	2A	*	74	4A	J	106	6A	j
11	0B	[VERTICAL TAB]	43	2B	+	75	4B	K	107	6B	k
12	0C	[FORM FEED]	44	2C	,	76	4C	L	108	6C	l
13	0D	[CARRIAGE RETURN]	45	2D	-	77	4D	M	109	6D	m
14	0E	[SHIFT OUT]	46	2E	.	78	4E	N	110	6E	n
15	0F	[SHIFT IN]	47	2F	/	79	4F	O	111	6F	o
16	10	[DATA LINK ESCAPE]	48	30	0	80	50	P	112	70	p
17	11	[DEVICE CONTROL 1]	49	31	1	81	51	Q	113	71	q
18	12	[DEVICE CONTROL 2]	50	32	2	82	52	R	114	72	r
19	13	[DEVICE CONTROL 3]	51	33	3	83	53	S	115	73	s
20	14	[DEVICE CONTROL 4]	52	34	4	84	54	T	116	74	t
21	15	[NEGATIVE ACKNOWLEDGE]	53	35	5	85	55	U	117	75	u
22	16	[SYNCHRONOUS IDLE]	54	36	6	86	56	V	118	76	v
23	17	[END OF TRANS. BLOCK]	55	37	7	87	57	W	119	77	w
24	18	[CANCEL]	56	38	8	88	58	X	120	78	x
25	19	[END OF MEDIUM]	57	39	9	89	59	Y	121	79	y
26	1A	[SUBSTITUTE]	58	3A	:	90	5A	Z	122	7A	z
27	1B	[ESCAPE]	59	3B	;	91	5B	[	123	7B	{
28	1C	[FILE SEPARATOR]	60	3C	<	92	5C	\	124	7C	
29	1D	[GROUP SEPARATOR]	61	3D	=	93	5D	]	125	7D	}
30	1E	[RECORD SEPARATOR]	62	3E	>	94	5E	^	126	7E	~
31	1F	[UNIT SEPARATOR]	63	3F	?	95	5F	_	127	7F	[DEL]

BCD – BCD stands for Binary Coded Decimal. It is normally used to break up a normal base 10 number into its digits. Each digit is routed to a different place. Each digit is directly converted to a binary value. **THIS IS NOT THE SAME AS CONVERTING A BASE 10 VALUE INTO A BASE 2 VALUE!**

For example: You may have a value such as 925, but you need to display it on a seven-segment display. But each segment can only display one digit. So, the original value needs to be broken up into three unique values and each value converted into binary.



This can also be done in reverse. There are “thumb wheels” that let you dial in a number one digit at a time. These digits make up a BCD value. The value needs to be converted to base 10 before it can be use.



Gray Code – This code came out of motors that needed to keep track of the motor's absolute position. The motor has an encoder disk on the back that had holes in it or was reflective. Gray code is just a list of binary numbers that only change by one bit at a time.

0 0 0
0 0 1
0 1 1
0 1 0
1 1 0
1 1 1
1 0 1
1 0 0

